

## STEP II, 2015, Q3

- 3 Three rods have lengths  $a$ ,  $b$  and  $c$ , where  $a < b < c$ . The three rods can be made into a triangle (possibly of zero area) if  $a + b \geq c$ .

Let  $T_n$  be the number of triangles that can be made with three rods chosen from  $n$  rods of lengths  $1, 2, 3, \dots, n$  (where  $n \geq 3$ ). Show that  $T_8 - T_7 = 2 + 4 + 6$  and evaluate  $T_8 - T_6$ . Write down expressions for  $T_{2m} - T_{2m-1}$  and  $T_{2m} - T_{2m-2}$ .

Prove by induction that  $T_{2m} = \frac{1}{6}m(m-1)(4m+1)$ , and find the corresponding result for an odd number of rods.



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