STEP II, 2015, Q2 MS

Question 2

As with all geometric questions a good diagram of the information given makes the solution to this question much easier to reach. The first result in this question follows from an application of the sine rule with applications of the relevant formulae for $\sin(A+B)$ and the double angle formulae. From a diagram of the triangle it should then be an easy application of trigonometry to show that $DE = \frac{x}{2}$. There are a number of different methods for establishing that FC trisects the angle ACB – one method is to show that $\sin(\angle FCE) = \frac{1}{2}$, following which it is relatively straightforward to work out the sizes of angles ACB and ACF in terms of α and show that they must satisfy the correct relationship.

Question 2

	$\angle ACB = \pi - 3\alpha$	**B1**
	$\frac{AB}{\sin(\pi - 3\alpha)} = \frac{x}{\sin\alpha}$	**M1 A1**
	$\sin(\pi - 3\alpha) = \sin 3\alpha$	**B1**
	$\therefore AB = \frac{x\sin 3\alpha}{\sin\alpha}$	
	$= \frac{x(\sin\alpha\cos 2\alpha + \cos\alpha\sin 2\alpha)}{\sin\alpha}$	**M1**
	$= \frac{x(\sin\alpha \times (1 - 2\sin^2\alpha) + \cos\alpha \times 2\sin\alpha\cos\alpha)}{\sin\alpha}$	**M1 M1**
	$AB = (3 - 4\sin^2\alpha)x$	**A1**
	$DE = AB - BE - AD$ (or $DE = DB - BE$)	**B1**
	$DE = AB - BE - \frac{1}{2}AB = \frac{1}{2}AB - BE$	
	$DE = \frac{x}{2}(3 - 4\sin^2\alpha) - x\cos 2\alpha$	**B1 B1**
	$DE = \frac{x}{2}\left((3 - 4\sin^2\alpha) - 2(1 - 2\sin^2\alpha)\right) = \frac{x}{2}$	**M1 M1 A1**
	$\sin(\angle FCE) = \frac{DE}{x} = \frac{1}{2}$, so $\angle FCE = \frac{\pi}{6}$	**B1 M1 A1**
	Therefore $\angle ACF = \pi - 3\alpha - \left(\frac{\pi}{2} - 2\alpha\right) - \frac{\pi}{6} = \frac{\pi}{3} - \alpha$	**M1 M1**
	$\angle ACF = \frac{1}{3}(\pi - 3\alpha) = \frac{1}{3}\angle ACB$ So FC trisects the angle ACB	**A1**