

## STEP II, 2015, Q12 MS

### Question 12

For the first part, note that  $A$  can only win the game if the first two tosses result in heads, since once there has been a tail,  $B$  will win as soon as two consecutive heads have been tossed and  $A$  cannot win until there have been two consecutive heads and one further toss. In the second part, note that this logic still applies to the game for  $A$  and similar reasoning can be applied to the game for  $C$ . For the other two players switching heads and tails in any sequence that results in a win for  $B$  will give a sequence that results in a win for  $D$ , and vice versa, so the probabilities must be equal. Since only sequences which alternate between heads and tails forever (and the probabilities of such sequences tend to zero as the lengths of the sequences increase) the probabilities must both also be  $\frac{1}{4}$ .

For the final part, note that  $C$  must win if the first two tosses are TT. Since only the previous two tosses are important in determining what could happen on the next toss, each case can be analysed by a tree diagram which shows the outcomes after one further toss.

For example, following HT:

- 
- H gives the position if the last two tosses were TH, and so a probability of winning of  $q$ ,
  - T gives the position if the last two tosses were TT and so a probability of winning of 1.

The total probability is therefore  $\frac{1}{2}q + \frac{1}{2}$ , but this must also be equal to  $p$ .

This yields three equations in the three unknowns which allows all of the individual probabilities to be calculated. Once this is done the overall probability can be calculated.



# NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to [NextStepMaths.com](http://NextStepMaths.com)

**Question 12**

(i)	If a tail occurs then player <i>B</i> must always win before <i>A</i> can achieve the sequence required. Therefore the only way for <i>A</i> to win is if both of the first two tosses are heads.	<b>B1</b>
	After the first two tosses are heads it does not matter if more tosses result in heads as the first time tails occurs <i>A</i> will win.	<b>B1</b>
	The probability that <i>A</i> wins is therefore $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$	<b>B1</b>
(ii)	As before, after <i>HH</i> , only <i>A</i> can win.	<b>B1</b>
	Similarly, after <i>TT</i> , only <i>C</i> can win.	<b>B1</b>
	In all other cases for the first two tosses only <i>B</i> and <i>D</i> will be able to win.	<b>M1</b>
	The probabilities for <i>B</i> and <i>D</i> to win must be equal.	<b>M1</b>
	The probability of winning is $\frac{1}{4}$ for all of the players.	<b>A1</b>
(iii)	If the first two tosses are <i>TT</i> then <i>C</i> must win (as soon as a <i>H</i> occurs), so the probability is 1.	<b>B1</b>
	After <i>HT</i> : <i>C</i> must win if the next toss is a <i>T</i> as <i>B</i> needs two <i>H</i> s to win, but <i>C</i> will win the next time an <i>H</i> occurs.	<b>M1</b>
	If the next toss is <i>H</i> , then the position is as if the first two tosses had been <i>TH</i> , and so the probability that <i>C</i> wins from this point is <i>q</i> .	<b>M1</b>
	Therefore, $p = \frac{1}{2} \times 1 + \frac{1}{2} \times q$	<b>A1</b>
	After <i>HH</i> : If the next toss is <i>H</i> then <i>C</i> will win with probability <i>r</i> . If the next toss is <i>T</i> then <i>C</i> will win with probability <i>p</i> .	<b>M1</b>
	Therefore $r = \frac{1}{2}r + \frac{1}{2}p$ , and so $p = r$ .	<b>A1</b>
	After <i>TH</i> : If the next toss is <i>H</i> then player <i>B</i> wins immediately. If the next toss is <i>T</i> then <i>C</i> will win with probability <i>p</i> .	<b>M1</b>
	Therefore $q = \frac{1}{2}p$ .	<b>A1</b>
	Solving the two equations in <i>p</i> and <i>q</i> , gives $p = \frac{2}{3}$ , $q = \frac{1}{3}$	
	From the third equation $r = \frac{2}{3}$	<b>M1 A1</b>
	The probability that <i>C</i> wins is $\frac{1}{4} \left( 1 + \frac{2}{3} + \frac{1}{3} + \frac{2}{3} \right) = \frac{2}{3}$	<b>M1 A1</b>



# NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to [NextStepMaths.com](https://www.NextStepMaths.com)