STEP II, 2015, Q10 MS

Question 10

The component of the velocity of the particle in the direction of the string at any moment must be equal to V, which leads to $V\ cosec\ \theta$ as the speed of the particle along the floor. Alternatively, introduce a variable to represent the length of string still in the room or the height of the room and then differentiate x, the distance of the particle from the point directly beneath the hole, with respect to time. The length of the string (to the hole in the ceiling) is decreasing at a rate of $V\ ms^{-1}$, which then allows the introduced variable to be eliminated to reach an expression for the speed of the particle.

Differentiation of the speed of the particle allows the acceleration to be calculated. Finally, note that the particle will remain on the floor as long as the vertical component of the tension is less than the weight of the particle and then the point at which the particle leaves the floor can be identified.

Question 10

	If the length of string from the hole at any moment is l, then $\frac{dl}{dt} = -V$.	**B1**
	The distance, x, from the point beneath the hole satisfies, $h^2 + x^2 = l^2$.	**B1**
	Therefore $\frac{dx}{dt} = \frac{d}{dt}\left((l^2 - h^2)^{\frac{1}{2}}\right) = \frac{1}{2}(l^2 - h^2)^{-\frac{1}{2}} \times 2l\frac{dl}{dt}$.	**M1 A1**
	$\frac{dx}{dt} = -lV(l^2 - h^2)^{-\frac{1}{2}} = -V \times \frac{l}{x}$, and $\frac{l}{x} = \operatorname{cosec}\theta$	**M1**
	Therefore, the speed of the particle is $V \operatorname{cosec}\theta$.	**A1**
	Acceleration: $\frac{d}{dt}(V\operatorname{cosec}\theta) = -V\operatorname{cosec}\theta\cot\theta \times \frac{d\theta}{dt}$	**M1 A1**
	$\sin\theta = \frac{x}{l}$, so $\cos\theta\frac{d\theta}{dt} = \frac{l(-V\operatorname{cosec}\theta) - l\sin\theta(-V)}{l^2} = \frac{V(\sin^2\theta - 1)}{l\sin\theta}$	**M1**
	Therefore $\frac{d\theta}{dt} = -\frac{V}{l}\cot\theta$	**A1**
	The acceleration is $\frac{V^2}{l\sin\theta}\cot^2\theta$	**M1**
	Since $l = h\sec\theta$, the acceleration can be written as $\frac{V^2}{h}\cot^3\theta$.	**M1 A1**
	Horizontally: $T\sin\theta = \frac{mV^2}{h}\cot^3\theta$, so $T = \frac{mV^2}{h}\cot^3\theta\operatorname{cosec}\theta$	**M1 M1** **A1**
	The particle will leave the floor when $T\cos\theta = mg$	**M1 A1**
	$\frac{mV^2}{h}\cot^4\theta = mg$ and so $\tan^4\theta = \frac{V^2}{gh}$	**M1 A1**