STEP II, 2014, Q9 MS

Question 9:

Once a diagram has been drawn the usual steps will lead to the required result:

Resolving vertically:

$$F + T\cos\theta = mg$$

Resolving horizontally:

$$T\sin\theta = R$$

Taking moments about A:

$$mg(a\cos\varphi + b\sin\varphi) = Td\sin(\theta + \varphi)$$

Limiting equilibrium, so $F = \mu R$:

$$\mu T\sin\theta + T\cos\theta = mg$$

Therefore:

$$Td\sin(\theta + \varphi) = T(\mu\sin\theta + \cos\theta)(a\cos\varphi + b\sin\varphi)$$

And so:

$$d\sin(\theta + \varphi) = (\mu\sin\theta + \cos\theta)(a\cos\varphi + b\sin\varphi)$$

If the frictional force were acting in the opposite direction, then the only change to the original equations would be the sign of F in the first equation. Therefore the final relationship will change to

$$d\sin(\theta + \varphi) = (-\mu\sin\theta + \cos\theta)(a\cos\varphi + b\sin\varphi)$$

For the final part, the first and third of the equations above can be used to show that

$$F = \frac{Td\sin(\theta + \varphi)}{a\cos\varphi + b\sin\varphi} - T\cos\theta$$

Since $F > 0$ if the frictional force is upwards, this then leads to the condition $d > \frac{a+b\tan\varphi}{\tan\theta+\tan\varphi}$. Since the string must be attached to the side AB, d cannot be bigger than $2b$, which leads to the final result of the question.