STEP II, 2014, Q8 MS

Question 8:

The coefficients from the binomial expansion should be easily written down. It can then be shown that

$$\frac{c_{r+1}}{c_r} = \frac{b(n-r)}{a(r+1)}$$

This will be greater than 1 (indicating that the value of c_r is increasing) while $b(n-r) > a(r+1)$, which simplifies to $r < \frac{nb-a}{a+b}$. Similarly, $\frac{c_{r+1}}{c_r} = 1$ if $r = \frac{nb-a}{a+b}$ and $\frac{c_{r+1}}{c_r} < 1$ if $r > \frac{nb-a}{a+b}$. Therefore the maximum value of c_r will be the first integer after $\frac{nb-a}{a+b}$ (and there will be two maximum values for c_r if $\frac{nb-a}{a+b}$ is an integer. The required inequality summarises this information.

In parts (i) and (ii) the values need to be substituted into the inequality. Where there are two possible values, it needs to be checked that they are equal before taking the higher if this has not been justified in the first case.

In part (iii) the greatest value will be achieved when the denominator takes the smallest possible value, therefore $a = 1$, and then in part (iv) the greatest value will be achieved by maximising the numerator. Since the maximum possible value of $G(n, a, b)$ is n, $b \geq n$ will achieve this maximum.