STEP II, 2014, Q6

6 By simplifying $\sin(r+\frac{1}{2})x - \sin(r-\frac{1}{2})x$ or otherwise show that, for $\sin\frac{1}{2}x \neq 0$,

$$\cos x + \cos 2x + \cdots + \cos nx = \frac{\sin(n+\frac{1}{2})x - \sin\frac{1}{2}x}{2\sin\frac{1}{2}x}\,.$$

The functions S_n, for $n = 1, 2, \ldots,$ are defined by

$$\mathrm{S}_n(x) = \sum_{r=1}^{n} \frac{1}{r}\sin rx \qquad (0 \leqslant x \leqslant \pi).$$

(i) Find the stationary points of $\mathrm{S}_2(x)$ for $0 \leqslant x \leqslant \pi$, and sketch this function.

(ii) Show that if $\mathrm{S}_n(x)$ has a stationary point at $x = x_0$, where $0 < x_0 < \pi$, then

$$\sin nx_0 = (1 - \cos nx_0)\tan\tfrac{1}{2}x_0$$

and hence that $\mathrm{S}_n(x_0) \geqslant \mathrm{S}_{n-1}(x_0)$. Deduce that if $\mathrm{S}_{n-1}(x) > 0$ for all x in the interval $0 < x < \pi$, then $\mathrm{S}_n(x) > 0$ for all x in this interval.

(iii) Prove that $\mathrm{S}_n(x) \geqslant 0$ for $n \geqslant 1$ and $0 \leqslant x \leqslant \pi$.

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