STEP II, 2014, Q5 MS

Question 5:

Using the substitution $y = xu$, the differential equation can be simplified to

$$x\frac{du}{dx} = \frac{1 + 4u - u^2}{u - 2}$$

This can be solved by separating the variables after which making the substitution $u = \frac{y}{x}$ and substituting the point on the curve gives the required quadratic in x and y.

In part (ii), $\frac{dY}{dX}$ can be shown to be equal to $\frac{dy}{dx}$. The values of a and b need to be chosen so that the right hand side of the differential equation has no constant terms in the numerator or denominator. This leads to the simultaneous equations:

$$a - 2b - 4 = 0$$

$$2a + b - 3 = 0$$

Solving these and substituting the values into the differential equation gives $\frac{dY}{dX} = \frac{X-2Y}{2X+Y}$, and so

$$\frac{dX}{dY} = \frac{2X + Y}{X - 2Y}$$

This is the same differential equation as in part (i), with $x = Y$ and $y = X$. Most of the solution in part (i) can therefore be applied, but the point on the curve is different, so the constant in the final solution will need to be calculated for this case.