# STEP II, 2014, Q4 MS

**Question 4:**

In part (i), if the required integral is called $I$ then the given substitution leads to an integral which can be shown to be equal to $-I$. This means that $2I = 0$ and so $I = 0$.

In part (ii), once the substitution has been completed, the integral will simplify to $\int_{1/b}^{b} \frac{\arctan\frac{1}{u}}{u} du$. Since $\arctan x + \arctan\left(\frac{1}{x}\right) = \frac{\pi}{2}$ the integral can be shown to be equal to $\frac{1}{2}\int_{1/b}^{b} \frac{\pi}{2x} dx$, which then simplifies to the required result.

In part (iii), making with the substitution in terms of $k$ and simplifying will show that the integral is equivalent to

$$\int_0^\infty \frac{ku^2}{(a^2u^2 + k^2)^2} du$$

Therefore choosing $k = a^2$, the integral can be simplified further to

$$\frac{1}{a^2}\int_0^\infty \frac{u^2}{(a^2 + u^2)^2} du = \frac{1}{a^2}\int_0^\infty \frac{1}{a^2 + u^2} du - \frac{1}{a^2}\int_0^\infty \frac{a^2}{(a^2 + u^2)^2} du$$

The result then follows by using the given value for $\int_0^\infty \frac{1}{a^2+x^2} dx$.