

STEP II, 2014, Q4

- 4 (i) By using the substitution $u = 1/x$, show that for $b > 0$

$$\int_{1/b}^b \frac{x \ln x}{(a^2 + x^2)(a^2 x^2 + 1)} dx = 0.$$

- (ii) By using the substitution $u = 1/x$, show that for $b > 0$,

$$\int_{1/b}^b \frac{\arctan x}{x} dx = \frac{\pi \ln b}{2}.$$

- (iii) By using the result $\int_0^\infty \frac{1}{a^2 + x^2} dx = \frac{\pi}{2a}$ (where $a > 0$), and a substitution of the form $u = k/x$, for suitable k , show that

$$\int_0^\infty \frac{1}{(a^2 + x^2)^2} dx = \frac{\pi}{4a^3} \quad (a > 0).$$



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