

STEP II, 2014, Q2

- 2 This question concerns the inequality

$$\int_0^{\pi} (f(x))^2 dx \leq \int_0^{\pi} (f'(x))^2 dx. \quad (*)$$

- (i) Show that (*) is satisfied in the case $f(x) = \sin nx$, where n is a positive integer.

Show by means of counterexamples that (*) is not necessarily satisfied if either $f(0) \neq 0$ or $f(\pi) \neq 0$.

- (ii) You may now assume that (*) is satisfied for any (differentiable) function f for which $f(0) = f(\pi) = 0$.

By setting $f(x) = ax^2 + bx + c$, where a , b and c are suitably chosen, show that $\pi^2 \leq 10$.

By setting $f(x) = p \sin \frac{1}{2}x + q \cos \frac{1}{2}x + r$, where p , q and r are suitably chosen, obtain another inequality for π .

Which of these inequalities leads to a better estimate for π^2 ?



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