

STEP II, 2014, Q13

- 13** A random number generator prints out a sequence of integers I_1, I_2, I_3, \dots . Each integer is independently equally likely to be any one of $1, 2, \dots, n$, where n is fixed. The random variable X takes the value r , where I_r is the first integer which is a repeat of some earlier integer.

Write down an expression for $P(X = 4)$.

- (i) Find an expression for $P(X = r)$, where $2 \leq r \leq n + 1$. Hence show that, for any positive integer n ,

$$\frac{1}{n} + \left(1 - \frac{1}{n}\right) \frac{2}{n} + \left(1 - \frac{1}{n}\right) \left(1 - \frac{2}{n}\right) \frac{3}{n} + \dots = 1.$$

- (ii) Write down an expression for $E(X)$. (You do not need to simplify it.)

- (iii) Write down an expression for $P(X \geq k)$.

- (iv) Show that, for any discrete random variable Y taking the values $1, 2, \dots, N$,

$$E(Y) = \sum_{k=1}^N P(Y \geq k).$$

Hence show that, for any positive integer n ,

$$\left(1 - \frac{1^2}{n}\right) + \left(1 - \frac{1}{n}\right) \left(1 - \frac{2^2}{n}\right) + \left(1 - \frac{1}{n}\right) \left(1 - \frac{2}{n}\right) \left(1 - \frac{3^2}{n}\right) + \dots = 0.$$



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