STEP II, 2014, Q12 MS

Question 12:

The required probability in the first part is given by

$$\frac{P(t < T < t + \delta t)}{P(T > t)} = \frac{F(t + \delta t) - F(t)}{1 - F(t)}$$

In the case of small values of δt, $F(t + \delta t) - F(t) \approx f(t)\delta t$, which leads to the correct probability.

In part (ii), differentiation gives $f(t) = \frac{1}{a}$, and substituting into the definition of the hazard function gives $h(t) = \frac{1}{a-t}$. Both graphs are simple to sketch.

In part (iii), using the definition of the hazard function gives $\frac{F\prime(t)}{1-F(t)} = \frac{1}{t}$. Integrating gives $-\ln|1 - F(t)| = \ln|kt|$, and so the probability density function can be found by rearranging to find $F(t)$ and then differentiating.

A similar method in part (iv) shows that if $h(t)$ is of the form stated then $f(t)$ will be of the given form. Similarly, if $f(t)$ has the given form then $h(t)$ can be shown to have the form stated.

In part (v), a differential equation can again be written using the definition of the hazard function and this can again be solved by integrating both sides with respect to t.