STEP II, 2014, Q11 MS

Question 11:

A diagram shows that the coordinates of P are $(x + (L - x)\sin\alpha\,, -(L - x)\cos\alpha)$

Therefore, by differentiating the y-coordinate of P shows that the vertical acceleration of P is $\ddot{x}\cos\alpha$ and applying Newton's Second Law gives

$$T\cos\alpha - kmg = km\ddot{x}\cos\alpha$$

A similar method for the horizontal motion of P and R gives the two equations

$$T\sin\alpha = -km(1 - \sin\alpha)\ddot{x}$$

$$T - T\sin\alpha = -m\ddot{x}$$

For part (ii), eliminating T from the last two equations gives the required relationship. A sketch of the graph of $y = \frac{x}{(1-x)^2}$ will then show that for any value of k there is a possible value between 0 and 1 for $\sin\alpha$.

In part (iii), elimination of T from the two equations formed by considering the motion of P gives the required result.