# STEP II, 2014, Q11

**11** A small smooth ring $R$ of mass $m$ is free to slide on a fixed smooth horizontal rail. A light inextensible string of length $L$ is attached to one end, $O$, of the rail. The string passes through the ring, and a particle $P$ of mass $km$ (where $k > 0$) is attached to its other end; this part of the string hangs at an acute angle $\alpha$ to the vertical and it is given that $\alpha$ is constant in the motion.

Let $x$ be the distance between $O$ and the ring. Taking the $y$-axis to be vertically upwards, write down the Cartesian coordinates of $P$ relative to $O$ in terms of $x$, $L$ and $\alpha$.

**(i)** By considering the vertical component of the equation of motion of $P$, show that

$$km\ddot{x}\cos\alpha = T\cos\alpha - kmg\,,$$

where $T$ is the tension in the string. Obtain two similar equations relating to the horizontal components of the equations of motion of $P$ and $R$.

**(ii)** Show that $\dfrac{\sin\alpha}{(1-\sin\alpha)^2} = k$, and deduce, by means of a sketch or otherwise, that motion with $\alpha$ constant is possible for all values of $k$.

**(iii)** Show that $\ddot{x} = -g\tan\alpha\,.$