

STEP II, 2014, Q10 MS

Question 10:

Consideration of the motion horizontally and vertically and eliminating the time variable leads to a Cartesian equation for the trajectory:

$$y = \lambda x - \frac{gx^2}{2u^2}(1 + \lambda^2)$$

The maximum value can be found either by differentiation or by completing the square. Completing the square gives:

$$y = -\frac{gx^2}{2u^2}\left(\lambda - \frac{u^2}{gx}\right) + \frac{u^2}{2g} - \frac{gx^2}{2u^2}$$

Which shows that $Y = \frac{u^2}{2g} - \frac{gx^2}{2u^2}$. If this graph is sketched then the region bounded by the graph and the axes will represent all the points that can be reached.

The maximum achievable distance must lie on the curve and the distance, d , of a point on the curve can be shown to satisfy $d^2 = \left(\frac{u^2}{2g} + \frac{gx^2}{2u^2}\right)^2$, which must be maximised when x takes the maximum value possible.



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