STEP II, 2013, Q8 MS

Question 8.

Begin by calculating the largest area of a rectangle with a given width and then maximize this function as the width of the rectangle is varied. The definition of x_0 can be reached by setting the derivative of the area function to 0.

The definition of g involves the differentiation of an integral of f which uses the variable t as the upper limit. The derivative of $tg(t)$ is therefore $f(t)$. The next statement relates the area bounded by the curve and the line $y = f(t)$ with the area of the largest rectangle with edges parallel to the axes that can fit into that space, so the first area must be greater and since that integral is equal to $tg(t) - tf(t)$ the result that follows is easily deduced.

The final part of the question involves finding expressions for $A_0(t)$ and $g(t)$ and then simplifying the relationship established at the end of part (ii).