

STEP II, 2013, Q7

- 7 (i) Write down a solution of the equation

$$x^2 - 2y^2 = 1, \quad (*)$$

for which x and y are non-negative integers.

Show that, if $x = p$, $y = q$ is a solution of (*), then so also is $x = 3p + 4q$, $y = 2p + 3q$. Hence find two solutions of (*) for which x is a positive odd integer and y is a positive even integer.

- (ii) Show that, if x is an odd integer and y is an even integer, (*) can be written in the form

$$n^2 = \frac{1}{2}m(m + 1),$$

where m and n are integers.

- (iii) The positive integers a , b and c satisfy

$$b^3 = c^4 - a^2,$$

where b is a prime number. Express a and c^2 in terms of b in the two cases that arise.

Find a solution of $a^2 + b^3 = c^4$, where a , b and c are positive integers but b is not prime.



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