

STEP II, 2013, Q5

- 5 (i) A function $f(x)$ satisfies $f(x) = f(1-x)$ for all x . Show, by differentiating with respect to x , that $f'(\frac{1}{2}) = 0$. If, in addition, $f(x) = f(\frac{1}{x})$ for all (non-zero) x , show that $f'(-1) = 0$ and that $f'(2) = 0$.

- (ii) The function f is defined, for $x \neq 0$ and $x \neq 1$, by

$$f(x) = \frac{(x^2 - x + 1)^3}{(x^2 - x)^2}.$$

Show that $f(x) = f(\frac{1}{x})$ and $f(x) = f(1-x)$.

Given that it has exactly three stationary points, sketch the curve $y = f(x)$.

- (iii) Hence, or otherwise, find all the roots of the equation $f(x) = \frac{27}{4}$ and state the ranges of values of x for which $f(x) > \frac{27}{4}$.

Find also all the roots of the equation $f(x) = \frac{343}{36}$ and state the ranges of values of x for which $f(x) > \frac{343}{36}$.



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