

STEP II, 2013, Q4

- 4 The line passing through the point $(a, 0)$ with gradient b intersects the circle of unit radius centred at the origin at P and Q , and M is the midpoint of the chord PQ . Find the coordinates of M in terms of a and b .
- (i) Suppose b is fixed and positive. As a varies, M traces out a curve (the *locus* of M). Show that $x = -by$ on this curve. Given that a varies with $-1 \leq a \leq 1$, show that the locus is a line segment of length $2b/(1 + b^2)^{\frac{1}{2}}$. Give a sketch showing the locus and the unit circle.
- (ii) Find the locus of M in the following cases, giving in each case its cartesian equation, describing it geometrically and sketching it in relation to the unit circle:
- (a) a is fixed with $0 < a < 1$, and b varies with $-\infty < b < \infty$;
- (b) $ab = 1$, and b varies with $0 < b \leq 1$.



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