# STEP II, 2013, Q3 MS

Question 3.

For it to be possible for the cubic to have three real roots it must have two stationary points. Since the coefficient of $x^3$ is positive it must have a specific shape. A sketch will show that only the two cases given will result in an intercept with the y-axis at a negative value.

In order for the cubic in part (ii) to have three positive roots, both of the turning points must be at positive values of $x$. Differentiation will allow most of the results to be established. The condition that $c < 0$ is needed to ensure that the leftmost root is also positive.

The condition $ab < 0$ implies that there must be a turning point at a positive value of $x$. The shape of the graph is as in part (i), but this time the intersection with the y-axis is at a positive value. This is sufficient to deduce the signs of the roots.

For part (iv) it is easiest to note that changing the value of $c$ does not (as long as $c$ remains negative) change whether or not the conditions of (*) are met. As this represents a vertical translation of the graph any example of a case satisfying (*) can be used to create an answer for this part by translating the graph sufficiently far downwards.