

STEP II, 2013, Q3

3 (i) Given that the cubic equation $x^3 + 3ax^2 + 3bx + c = 0$ has three distinct real roots and $c < 0$, show with the help of sketches that either exactly one of the roots is positive or all three of the roots are positive.

(ii) Given that the equation $x^3 + 3ax^2 + 3bx + c = 0$ has three distinct real positive roots show that

$$a^2 > b > 0, \quad a < 0, \quad c < 0. \quad (*)$$

[**Hint:** Consider the turning points.]

(iii) Given that the equation $x^3 + 3ax^2 + 3bx + c = 0$ has three distinct real roots and that

$$ab < 0, \quad c > 0,$$

determine, with the help of sketches, the signs of the roots.

(iv) Show by means of an explicit example (giving values for a , b and c) that it is possible for the conditions (*) to be satisfied even though the corresponding cubic equation has only one real root.



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to

NextStepMaths.com