

STEP II, 2013, Q2 MS

Question 2.

The obvious substitution in the first part leads easily to the required result. It should then be easy to establish the second result by making the integral into the sum of two integrals and noting that taking out a common factor leaves $(1 - x) + x$ to be simplified. Integration by parts will lead to the next result after which taking out one of the factors of $(1 - x)$ will allow the integral to be split into a difference of two integrals.

The result in part (ii) is most easily proved by induction. It is necessary to fill in the gap in the factorial on the denominator by multiplying both the numerator and denominator by the missing even number. In alternative approaches, it needs to be remembered that the product of the even numbers up to and including $2n$ can be written as $2^n n!$

The final part is a straightforward substitution, although care needs to be taken with the signs. The final result can be obtained using the relationship established in part (i) as none of the reasoning requires n to be an integer.



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