# STEP II, 2013, Q13 MS

Question 13.

An alternating run of length 1 must be two results showing the same side of the coin. It is then easy to show that the probability is as given. Similarly a straight run of length 1 must be two different results (in either order) and so the probability can again be calculated easily. The terms involved are those in the expansion of $(p \pm q)^2$ and so starting with the statement that $(p - q)^2 \geq 0$ then relationship between the two probabilities can be established.

An alternating run of length 2 must be one result followed by the other one twice, while a straight run of length 2 must be two identical results followed by the other one. They will therefore be calculated by the same sums (with the products in a different order each time) so the probabilities must be equal. By considering the ways in which runs of length 3 can be obtained it is clear that these two probabilities must also be equal.

An alternating run of length $2n$ must be $n$ of each of the two possibilities followed by a repeat of whichever came last. A straight run of length $2n$ must be $2n$ of one of the possibilities followed by 1 of the other. Taking the difference between these two probabilities gives an expression which can be seen to always have the same sign, which will determine which probability is greater. A similar method will also work for the final case.