

STEP II, 2013, Q12

- 12 The random variable U has a Poisson distribution with parameter λ . The random variables X and Y are defined as follows.

$$X = \begin{cases} U & \text{if } U \text{ is } 1, 3, 5, 7, \dots \\ 0 & \text{otherwise} \end{cases}$$

$$Y = \begin{cases} U & \text{if } U \text{ is } 2, 4, 6, 8, \dots \\ 0 & \text{otherwise} \end{cases}$$

- (i) Find $E(X)$ and $E(Y)$ in terms of λ , α and β , where

$$\alpha = 1 + \frac{\lambda^2}{2!} + \frac{\lambda^4}{4!} + \dots \quad \text{and} \quad \beta = \frac{\lambda}{1!} + \frac{\lambda^3}{3!} + \frac{\lambda^5}{5!} + \dots$$

- (ii) Show that

$$\text{Var}(X) = \frac{\lambda\alpha + \lambda^2\beta}{\alpha + \beta} - \frac{\lambda^2\alpha^2}{(\alpha + \beta)^2}$$

and obtain the corresponding expression for $\text{Var}(Y)$. Are there any non-zero values of λ for which $\text{Var}(X) + \text{Var}(Y) = \text{Var}(X + Y)$?



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