

STEP II, 2013, Q1

- 1 (i) Find the value of m for which the line $y = mx$ touches the curve $y = \ln x$.
If instead the line intersects the curve when $x = a$ and $x = b$, where $a < b$, show that $a^b = b^a$. Show by means of a sketch that $a < e < b$.
- (ii) The line $y = mx + c$, where $c > 0$, intersects the curve $y = \ln x$ when $x = p$ and $x = q$, where $p < q$. Show by means of a sketch, or otherwise, that $p^q > q^p$.
- (iii) Show by means of a sketch that the straight line through the points $(p, \ln p)$ and $(q, \ln q)$, where $e \leq p < q$, intersects the y -axis at a positive value of y . Which is greater, π^e or e^π ?
- (iv) Show, using a sketch or otherwise, that if $0 < p < q$ and $\frac{\ln q - \ln p}{q - p} = e^{-1}$, then $q^p > p^q$.



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