# STEP II, 2013, Q1

**1** **(i)** Find the value of $m$ for which the line $y = mx$ touches the curve $y = \ln x$.

If instead the line intersects the curve when $x = a$ and $x = b$, where $a < b$, show that $a^b = b^a$. Show by means of a sketch that $a < \mathrm{e} < b$.

**(ii)** The line $y = mx + c$, where $c > 0$, intersects the curve $y = \ln x$ when $x = p$ and $x = q$, where $p < q$. Show by means of a sketch, or otherwise, that $p^q > q^p$.

**(iii)** Show by means of a sketch that the straight line through the points $(p, \ln p)$ and $(q, \ln q)$, where $\mathrm{e} \leqslant p < q$, intersects the $y$-axis at a positive value of $y$. Which is greater, $\pi^{\mathrm{e}}$ or $\mathrm{e}^{\pi}$?

**(iv)** Show, using a sketch or otherwise, that if $0 < p < q$ and $\dfrac{\ln q - \ln p}{q - p} = \mathrm{e}^{-1}$, then $q^p > p^q$.