

## **STEP II, 2012 Q8**

- 8 The positive numbers  $\alpha$ ,  $\beta$  and  $q$  satisfy  $\beta - \alpha > q$ . Show that

$$\frac{\alpha^2 + \beta^2 - q^2}{\alpha\beta} - 2 > 0.$$

The sequence  $u_0, u_1, \dots$  is defined by  $u_0 = \alpha$ ,  $u_1 = \beta$  and

$$u_{n+1} = \frac{u_n^2 - q^2}{u_{n-1}} \quad (n \geq 1),$$

where  $\alpha$ ,  $\beta$  and  $q$  are given positive numbers (and  $\alpha$  and  $\beta$  are such that no term in the sequence is zero). Prove that  $u_n(u_n + u_{n+2}) = u_{n+1}(u_{n-1} + u_{n+1})$ . Prove also that

$$u_{n+1} - pu_n + u_{n-1} = 0$$

for some number  $p$  which you should express in terms of  $\alpha$ ,  $\beta$  and  $q$ .

Hence, or otherwise, show that if  $\beta > \alpha + q$ , the sequence is strictly increasing (that is,  $u_{n+1} - u_n > 0$  for all  $n$ ). Comment on the case  $\beta = \alpha + q$ .



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