

## STEP II, 2012 Q7

- 7 Three distinct points,  $X_1$ ,  $X_2$  and  $X_3$ , with position vectors  $\mathbf{x}_1$ ,  $\mathbf{x}_2$  and  $\mathbf{x}_3$  respectively, lie on a circle of radius 1 with its centre at the origin  $O$ . The point  $G$  has position vector  $\frac{1}{3}(\mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3)$ . The line through  $X_1$  and  $G$  meets the circle again at the point  $Y_1$  and the points  $Y_2$  and  $Y_3$  are defined correspondingly.

Given that  $\overrightarrow{GY_1} = -\lambda_1 \overrightarrow{GX_1}$ , where  $\lambda_1$  is a positive scalar, show that

$$\overrightarrow{OY_1} = \frac{1}{3}((1 - 2\lambda_1)\mathbf{x}_1 + (1 + \lambda_1)(\mathbf{x}_2 + \mathbf{x}_3))$$

and hence that

$$\lambda_1 = \frac{3 - \alpha - \beta - \gamma}{3 + \alpha - 2\beta - 2\gamma},$$

where  $\alpha = \mathbf{x}_2 \cdot \mathbf{x}_3$ ,  $\beta = \mathbf{x}_3 \cdot \mathbf{x}_1$  and  $\gamma = \mathbf{x}_1 \cdot \mathbf{x}_2$ .

Deduce that  $\frac{GX_1}{GY_1} + \frac{GX_2}{GY_2} + \frac{GX_3}{GY_3} = 3$ .



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