STEP II, 2012 Q6

6 A cyclic quadrilateral $ABCD$ has sides AB, BC, CD and DA of lengths a, b, c and d, respectively. The area of the quadrilateral is Q, and angle DAB is θ.

Find an expression for $\cos\theta$ in terms of a, b, c and d, and an expression for $\sin\theta$ in terms of a, b, c, d and Q. Hence show that

$$16Q^2 = 4(ad+bc)^2 - (a^2+d^2-b^2-c^2)^2\,,$$

and deduce that

$$Q^2 = (s-a)(s-b)(s-c)(s-d)\,,$$

where $s = \frac{1}{2}(a+b+c+d)$.

Deduce a formula for the area of a triangle with sides of length a, b and c.