

STEP II, 2012 Q3

- 3 Show that, for any function f (for which the integrals exist),

$$\int_0^{\infty} f(x + \sqrt{1+x^2}) dx = \frac{1}{2} \int_1^{\infty} \left(1 + \frac{1}{t^2}\right) f(t) dt.$$

Hence evaluate

$$\int_0^{\infty} \frac{1}{2x^2 + 1 + 2x\sqrt{x^2 + 1}} dx,$$

and, using the substitution $x = \tan \theta$,

$$\int_0^{\frac{1}{2}\pi} \frac{1}{(1 + \sin \theta)^3} d\theta.$$



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