

## STEP II, 2012, Q13 MS

### Question 13

Working with the distribution  $P_o(\lambda = k\pi y^2)$ ,  $P(\text{no supermarkets}) = e^{-k\pi y^2}$  and  $P(Y < y) = 1 - e^{-k\pi y^2}$ . Differentiating w.r.t.  $y$  to find the pdf of  $Y \Rightarrow f(y) = 2k\pi y e^{-k\pi y^2}$ , as given. Then

$$E(Y) = \int_0^{\infty} 2k\pi y^2 e^{-k\pi y^2} dy. \text{ Using } \textit{Integration by Parts} \text{ and writing } 2k\pi y^2 e^{-k\pi y^2} \text{ as } y \left( 2k\pi y e^{-k\pi y^2} \right)$$

gives  $E(Y) = \left[ y \left( e^{-k\pi y^2} \right) \right]_0^{\infty} + \int_0^{\infty} e^{-k\pi y^2} dy = 0 + \int_0^{\infty} e^{-k\pi y^2} dy$ . It is useful (but not essential) to use the

simplifying substitution  $x = y\sqrt{2k\pi}$  at this stage to get  $\frac{1}{\sqrt{2k\pi}} \int_0^{\infty} e^{-\frac{1}{2}x^2} dx = \frac{1}{\sqrt{2k\pi}} \sqrt{\frac{\pi}{2}} = \frac{1}{2\sqrt{k}}$  (by the given result, relating to the standard normal distribution's pdf, at the very beginning of the question).

Next,  $E(Y^2) = \int_0^{\infty} 2k\pi y^3 e^{-k\pi y^2} dy$ , and using *Integration by Parts* and, in a similar way to earlier,

$$\text{writing } 2k\pi y^3 e^{-k\pi y^2} \text{ as } y^2 \left( 2k\pi y e^{-k\pi y^2} \right), \quad E(Y^2) = \left[ y^2 \left( e^{-k\pi y^2} \right) \right]_0^{\infty} + \int_0^{\infty} 2y e^{-k\pi y^2} dy$$

$$= 0 + \frac{1}{k\pi} \int_0^{\infty} 2k\pi y e^{-k\pi y^2} dy = \frac{-1}{k\pi} \left[ e^{-k\pi y^2} \right]_0^{\infty} \quad (\text{using a previous result, or by substitution}) = \frac{1}{k\pi}$$

$$\Rightarrow \text{Var}(Y) = \frac{1}{k\pi} - \frac{1}{4k} = \frac{4 - \pi}{4k\pi}, \text{ the given answer, as required.}$$



# NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to

[NextStepMaths.com](http://NextStepMaths.com)