

STEP II, 2012 Q12

- 12 A modern villa has complicated lighting controls. In order for the light in the swimming pool to be on, a particular switch in the hallway must be on and a particular switch in the kitchen must be on. There are four identical switches in the hallway and four identical switches in the kitchen. Guests cannot tell whether the switches are on or off, or what they control.

Each Monday morning a guest arrives, and the switches in the hallway are either all on or all off. The probability that they are all on is p and the probability that they are all off is $1 - p$. The switches in the kitchen are each on or off, independently, with probability $\frac{1}{2}$.

- (i) On the first Monday, a guest presses one switch in the hallway at random and one switch in the kitchen at random. Find the probability that the swimming pool light is on at the end of this process. Show that the probability that the guest has pressed the swimming pool light switch in the hallway, given that the light is on at the end of the process, is $\frac{1-p}{1+2p}$.
- (ii) On each of seven Mondays, guests go through the above process independently of each other, and each time the swimming pool light is found to be on at the end of the process. Given that the most likely number of days on which the swimming pool light switch in the hallway was pressed is 3, show that $\frac{1}{4} < p < \frac{5}{14}$.



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