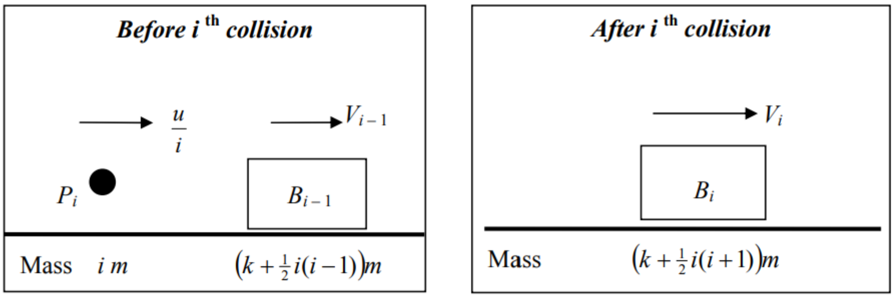
STEP II, 2012, Q11 MS

Question 11

Again, a diagram is really useful for helping put ones thoughts in order; also, we are going to have to consider what is going on generally (and not just "pattern-spot" our way up the line).



Using the principle of *Conservation of Linear Momentum*,

<u>CLM →</u> $\quad m\,u + M\,V_{i-1} = (M + im)\,V_i \qquad$ (NB $V_0 = 0$) leads to

$$V_1 = \frac{u}{k+1},\; V_2 = \frac{2u}{k+1+2},\; V_3 = \frac{3u}{k+1+2+3},\; \dots,\; V_n = \frac{nu}{k+\frac{1}{2}n(n+1)} = \frac{2nu}{2k+n(n+1)}.$$

Alternatively, <u>CLM → for *all* particles</u> gives $mu + 2m\left(\frac{u}{2}\right) + 3m\left(\frac{u}{3}\right) + \dots\dots + nm\left(\frac{u}{n}\right) = \left(k+\frac{1}{2}n(n+1)\right)mV$,

and rearranging for $V = V_n$ yields $V_n = \frac{2nu}{2k+n(n+1)}$.

The last collision occurs when $V_n \ge \frac{u}{n+1}$, i.e. $\frac{2nu}{N(N+1)+n(n+1)} \ge \frac{u}{n+1}$

$\Rightarrow\; 2n(n+1) \ge N(N+1) + n(n+1) \;\Rightarrow\; n(n+1) \ge N(N+1) \;\Rightarrow$ there are N collisions.

Now, the total KE of all the P_i's is $\sum_{i=1}^{N} \frac{1}{2}(i\,m)\left(\frac{u}{i}\right)^2 = \frac{1}{2}mu^2 \sum_{i=1}^{N} \frac{1}{i}$.

The final KE of the block is $\frac{1}{2}N(N+1)mV_N^{\,2} = \frac{1}{2}N(N+1)m\left(\frac{u}{N+1}\right)^2 = \frac{1}{2}mu^2\left(\frac{N}{N+1}\right)$.

Therefore, the loss in KE is the difference: $\frac{1}{2}mu^2 \sum_{i=1}^{N} \frac{1}{i} - \frac{1}{2}mu^2\left(\frac{N}{N+1}\right)$.

Since $\frac{N}{N+1} = 1 - \frac{1}{N+1}$, the loss in KE is $\frac{1}{2}mu^2\left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{N} - 1 + \frac{1}{N+1}\right) = \frac{1}{2}mu^2 \sum_{i=2}^{N+1}\left(\frac{1}{i}\right)$.