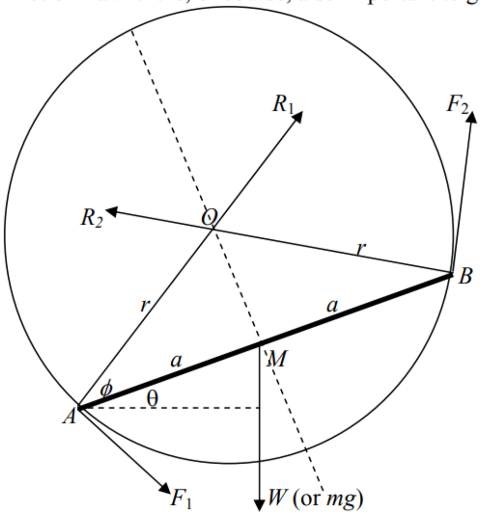
STEP II, 2012, Q10 MS

Question 10

As with many statics problems, a good diagram is essential to successful progress. Then there are relatively few mechanical principles to be applied ... resolving (twice), taking moments, and the standard "Friction Law". It is, of course, also important to get the angles right.



Taking moments about M :

$R_1\, a \sin\phi = R_2\, a \sin\phi + F_1\, a \cos\phi + F_2\, a \cos\phi$

Using the Friction Law : $F_1 = \mu R_1$ and $F_2 = \mu R_2$

Dividing by $\cos\phi$ and re-arranging

$R_1 \tan\phi = R_2 \tan\phi + \mu R_1 + \mu R_2$

$\Rightarrow (R_1 - R_2)\tan\phi = \mu(R_1 + R_2)$

For the second part, it seems likely that we will have to resolve twice (not having yet used this particular set of tools), though we could take moments about some other point in place of one resolution. There is also the question of which directions to resolve in – here, it should be clear very quickly that "horizontally and vertically" will only yield some very messy results.

Moments about O : $\mu(R_1 - R_2)\, r = W\, r \sin\phi \sin\theta$

Resolving // AB : $(R_1 - R_2)\cos\phi + \mu(R_1 + R_2)\sin\phi = W \sin\theta$

(Give one **A1** here if all correct apart from a – sign)

Resolving $\perp^r$ AB : $(R_1 + R_2)\sin\phi - \mu(R_1 - R_2)\cos\phi = W\cos\theta$

Note that only two of these are actually required, but it may be easier to write them all down first and *then* decide which two are best used.

Dividing these last two eqns. $\Rightarrow \tan\theta = \dfrac{(R_1 - R_2)\cos\phi + \mu(R_1 + R_2)\sin\phi}{(R_1 + R_2)\sin\phi - \mu(R_1 - R_2)\cos\phi}$

Using first result, $\mu(R_1 + R_2) = (R_1 - R_2)\tan\phi \Rightarrow \tan\theta = \dfrac{(R_1 - R_2)\cos\phi + (R_1 - R_2)\tan\phi\sin\phi}{(R_1 - R_2)\dfrac{\tan\phi}{\mu}\sin\phi - \mu(R_1 - R_2)\cos\phi}$

$\Rightarrow \tan\theta = \dfrac{\cos\phi + \tan\phi\sin\phi}{\dfrac{\tan\phi}{\mu}\sin\phi - \mu\cos\phi}$. (There is no need to note that $R_1 \neq R_2$ for then the rod would hve to be positioned symmetrically in the cylinder.)

Multiplying throughout by $\mu\cos\phi \Rightarrow \tan\theta = \dfrac{\mu(\cos^2\phi + \sin^2\phi)}{\sin^2\phi - \mu^2\cos^2\phi} = \dfrac{\mu}{1 - \cos^2\phi - \mu^2\cos^2\phi}$ and, using

$\cos\phi = \dfrac{a}{r}$ gives $\tan\theta = \dfrac{\mu}{1 - \dfrac{a^2}{r^2} - \mu^2\left(\dfrac{a^2}{r^2}\right)} = \dfrac{\mu r^2}{r^2 - a^2(1 + \mu^2)}$.

Finally, $\tan\lambda = \mu = \left(\dfrac{R_1 - R_2}{R_1 + R_2}\right)\tan\phi$, from the first result, $< \tan\phi \Rightarrow \lambda < \phi$.