STEP II, 2011, Q9 MS

Q9 Collisions questions are always popular, as there are only two or three principles which are to be applied. It is, nonetheless, good practice to say what you are attempting to do. Also, a diagram, though not an essential requirement, is almost always a good idea, if only since it enables you to specify a direction which you are going to take to be the positive one, especially since velocity and momentum are vector quantities. Once these preliminaries have been set up, the rest is fairly easy. By *CLM*, $3mu = 2mV_A + mV_B$ and *NEL/NLR* gives $e.3u = V_B - V_A$. Solving these simultaneously for V_A and V_B yields $V_A = u(1 - e)$ and $V_B = u(1 + 2e)$.

Next, after its collision with the wall, B has speed $|f V_B|$ away from the wall.

For the second collision of A and B, by *CLM* (away from wall), $fmV_B - 2mV_A = 2mW_A - mW_B$, and *NEL/NLR* gives $W_A + W_B = e(V_A + f V_B)$. Subst^g. for V_A & V_B from before in *both* of these equations $\Rightarrow 2W_A - W_B = u\{f(1+2e) - 2(1-e)\}$ and $W_A + W_B = eu\{(1-e) + f(1+2e)\}$. Solving these simultaneously for W_A (not wanted) and W_B then gives $W_B = \frac{1}{3}u\{2(1-e^2) - f(1-4e^2)\}$, as required.

Noting that $1-4e^2$ can be negative, zero, or positive, it may be best (though not essential) to consider the possible cases separately:

if $e = \frac{1}{2}$, $W_B = \frac{1}{3}u\{2(\frac{3}{4}) - f(0)\} = \frac{1}{2}u > 0$;

if $\frac{1}{2} < e < 1$, $W_B = \frac{1}{3}u\{2(1-e^2) + f(4e^2 - 1)\} > 0$ for all e, f since each term in the bracket is > 0;

if $0 < e < \frac{1}{2}$, $1 - e^2 > \frac{3}{4}$ and $W_B > \frac{1}{3}u\{\frac{3}{2} - f(1-4e^2)\} > \frac{1}{3}u\{\frac{3}{2} - 1 \times 1\} > 0$.