

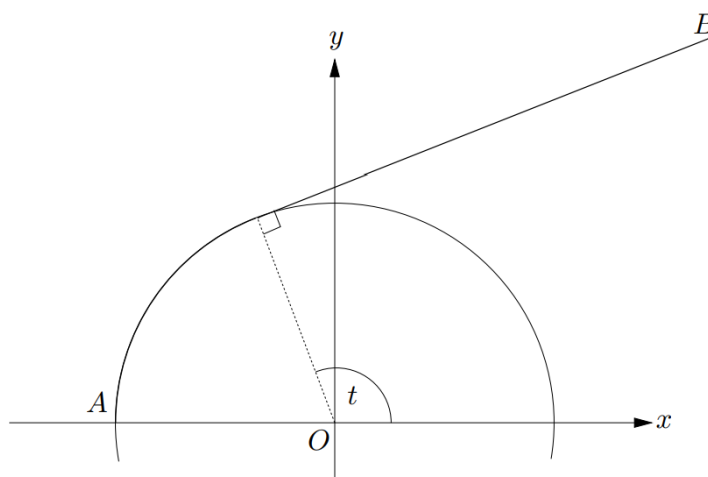
STEP II, 2011 Q8

- 8 The end A of an inextensible string AB of length π is attached to a point on the circumference of a fixed circle of unit radius and centre O . Initially the string is straight and tangent to the circle. The string is then wrapped round the circle until the end B comes into contact with the circle. The string remains taut during the motion, so that a section of the string is in contact with the circumference and the remaining section is straight.

Taking O to be the origin of cartesian coordinates with A at $(-1, 0)$ and B initially at $(-1, \pi)$, show that the curve described by B is given parametrically by

$$x = \cos t + t \sin t, \quad y = \sin t - t \cos t,$$

where t is the angle shown in the diagram.



Find the value, t_0 , of t for which x takes its maximum value on the curve, and sketch the curve.

Use the area integral $\int y \frac{dx}{dt} dt$ to find the area between the curve and the x axis for $\pi \geq t \geq t_0$.

Find the area swept out by the string (that is, the area between the curve described by B and the semicircle shown in the diagram).



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to

NextStepMaths.com