

## STEP II, 2011 Q7

7 The two sequences  $a_0, a_1, a_2, \dots$  and  $b_0, b_1, b_2, \dots$  have general terms

$$a_n = \lambda^n + \mu^n \quad \text{and} \quad b_n = \lambda^n - \mu^n,$$

respectively, where  $\lambda = 1 + \sqrt{2}$  and  $\mu = 1 - \sqrt{2}$ .

(i) Show that  $\sum_{r=0}^n b_r = -\sqrt{2} + \frac{1}{\sqrt{2}} a_{n+1}$ , and give a corresponding result for  $\sum_{r=0}^n a_r$ .

(ii) Show that, if  $n$  is odd,

$$\sum_{m=0}^{2n} \left( \sum_{r=0}^m a_r \right) = \frac{1}{2} b_{n+1}^2,$$

and give a corresponding result when  $n$  is even.

(iii) Show that, if  $n$  is even,

$$\left( \sum_{r=0}^n a_r \right)^2 - \sum_{r=0}^n a_{2r+1} = 2,$$

and give a corresponding result when  $n$  is odd.



# NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to

[NextStepMaths.com](http://NextStepMaths.com)