

## STEP II, 2011 Q6

6 For any given function  $f$ , let

$$I = \int [f'(x)]^2 [f(x)]^n dx, \quad (*)$$

where  $n$  is a positive integer. Show that, if  $f(x)$  satisfies  $f''(x) = kf(x)f'(x)$  for some constant  $k$ , then  $(*)$  can be integrated to obtain an expression for  $I$  in terms of  $f(x)$ ,  $f'(x)$ ,  $k$  and  $n$ .

(i) Verify your result in the case  $f(x) = \tan x$ . Hence find

$$\int \frac{\sin^4 x}{\cos^8 x} dx.$$

(ii) Find

$$\int \sec^2 x (\sec x + \tan x)^6 dx.$$



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