

STEP II, 2011 Q5

- 5 The points A and B have position vectors \mathbf{a} and \mathbf{b} with respect to an origin O , and O , A and B are non-collinear. The point C , with position vector \mathbf{c} , is the reflection of B in the line through O and A . Show that \mathbf{c} can be written in the form

$$\mathbf{c} = \lambda\mathbf{a} - \mathbf{b}$$

where $\lambda = \frac{2\mathbf{a}\cdot\mathbf{b}}{\mathbf{a}\cdot\mathbf{a}}$.

The point D , with position vector \mathbf{d} , is the reflection of C in the line through O and B . Show that \mathbf{d} can be written in the form

$$\mathbf{d} = \mu\mathbf{b} - \lambda\mathbf{a}$$

for some scalar μ to be determined.

Given that A , B and D are collinear, find the relationship between λ and μ . In the case $\lambda = -\frac{1}{2}$, determine the cosine of $\angle AOB$ and describe the relative positions of A , B and D .



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