

## STEP II, 2011 Q3

**3** In this question, you may assume without proof that any function  $f$  for which  $f'(x) \geq 0$  is *increasing*; that is,  $f(x_2) \geq f(x_1)$  if  $x_2 \geq x_1$ .

(i) (a) Let  $f(x) = \sin x - x \cos x$ . Show that  $f(x)$  is increasing for  $0 \leq x \leq \frac{1}{2}\pi$  and deduce that  $f(x) \geq 0$  for  $0 \leq x \leq \frac{1}{2}\pi$ .

(b) Given that  $\frac{d}{dx}(\arcsin x) \geq 1$  for  $0 \leq x < 1$ , show that

$$\arcsin x \geq x \quad (0 \leq x < 1).$$

(c) Let  $g(x) = x \operatorname{cosec} x$  for  $0 < x < \frac{1}{2}\pi$ . Show that  $g$  is increasing and deduce that

$$(\arcsin x) x^{-1} \geq x \operatorname{cosec} x \quad (0 < x < 1).$$

(ii) Given that  $\frac{d}{dx}(\arctan x) \leq 1$  for  $x \geq 0$ , show by considering the function  $x^{-1} \tan x$  that

$$(\tan x)(\arctan x) \geq x^2 \quad (0 < x < \frac{1}{2}\pi).$$



# NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to

[NextStepMaths.com](http://NextStepMaths.com)