

STEP II, 2011, Q2 EC

Q2 Personally, this was my favourite question, even though it was ultimately (marginally) deflected from its original purpose of expressing integers as sums of two rational cubes. Given that the question explicitly involves inequalities (which are, as a rule, never popular) and cubics rather than quadratics, it was slightly surprising to find that it was the most popular question on the paper. However, although the average score on the question was almost exactly 10, these two issues then turned out to be the biggest stumbling-blocks to a completely successful attempt as candidates progressed through the question, both in establishing the given inequalities and then in the use of them. In particular, it was noted that many candidates “proved” the given results by showing that they implied something else that was true, rather than by *deducing* them from something else known to be true; such logical flaws received little credit in terms of marks. The purpose of this preliminary work was to enable the candidates to whittle down the possibilities to a small, finite list and then provide them with some means of testing each possibility’s validity. This help was often ignored in favour of starting again. In general, though, part (i) was done reasonably well; as was (ii) by those who used (i)’s methodology as a template.

Only a very few candidates were bold enough to attempt (ii) successfully without any reference to (i)’s methods; indeed, this arithmetic approach was how the question was originally posed (as part (i), of course) before proceeding onto the algebra. Noting that the wording of the question does not demand any particular approach in order to find the required *two* solutions to the equation $x^3 + y^3 = 19z^3$, a reasonably confident arithmetician might easily note that

$$19 \times 2^3 = 152 = 3^3 + 5^3 \quad \text{and} \quad 19 \times 3^3 = 513 = 1^3 + 8^3$$

and it isn’t even necessary to look very far for *two* solutions. For 10 marks, this is what our transatlantic cousins would call “a steal”.



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