

STEP II, 2011 Q2

- 2 Write down the cubes of the integers $1, 2, \dots, 10$.

The positive integers x, y and z , where $x < y$, satisfy

$$x^3 + y^3 = kz^3, \quad (*)$$

where k is a given positive integer.

- (i) In the case $x + y = k$, show that

$$z^3 = k^2 - 3kx + 3x^2.$$

Deduce that $(4z^3 - k^2)/3$ is a perfect square and that $\frac{1}{4}k^2 \leq z^3 < k^2$.

Use these results to find a solution of (*) when $k = 20$.

- (ii) By considering the case $x + y = z^2$, find two solutions of (*) when $k = 19$.



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