STEP II, 2011, Q1 MS

Q1 **(i)** There are several routine features of a graph that one should look to consider on any curve-sketching question: key points, such as where the curve meets or cuts either of the coordinate axes, symmetries (and periodicities for trig. functions), asymptotes, and turning-points are the usual suspects. In this case, the given function involves square-roots as well, so the question of the domain of the function also comes into question. Considering all such things for $y = \sqrt{1-x} + \sqrt{3+x}$ should help you realise the following:

* the RHS is only defined for $-3 \le x \le 1$ (so the endpoints are at (–3, 2) and (1, 2));

* the graph is symmetric in the line $x = -1$, with its maximum at $\left(-1,\ 2\sqrt{2}\right)$; NB it must be a maximum since $2\sqrt{2} > 2$ so there is no need to resort to calculus to establish this;

* the curve is thus $\cap$ –shaped, and the gradient at the endpoints is infinite. This last point wasn't of great concern for the purposes of the question, so its mention was neither rewarded nor its lack penalised: however, this is easily determined by realising that each term in the RHS is of the form $X^{\frac{1}{2}}$, so their derivatives will be of the form $X^{-\frac{1}{2}}$ which, when evaluated at an endpoint will give one of them of the form $\frac{1}{0}$ symptomatic of an asymptote.

A quick sketch of $y = x + 1$ shows that there is only the one solution at $x = 1$.

(ii) Each side of this second equation represents an easily sketchable curve. Indeed, the RHS is essentially the same curve as in (i), but defined on the interval [–3, 3]. The LHS is merely a "horizontal" parabola, though only its upper half since the radix $\left(\sqrt{\ }\right)$ sign denotes the *positive* square-root. These curves again intersect only the once, when $x < 0$. Resorting to algebra … squaring, rearranging suitably and squaring again then yields a quadratic equation in x having one positive and one negative root.