STEP II, 2011 Q13

13 What property of a distribution is measured by its *skewness*?

(i) One measure of skewness, γ, is given by

$$\gamma = \frac{\mathrm{E}\big((X-\mu)^3\big)}{\sigma^3},$$

where μ and σ^2 are the mean and variance of the random variable X. Show that

$$\gamma = \frac{\mathrm{E}(X^3) - 3\mu\sigma^2 - \mu^3}{\sigma^3}.$$

The continuous random variable X has probability density function f where

$$\mathrm{f}(x) = \begin{cases} 2x & \text{for } 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise}. \end{cases}$$

Show that for this distribution $\gamma = -\dfrac{2\sqrt{2}}{5}$.

(ii) The *decile skewness*, D, of a distribution is defined by

$$D = \frac{\mathrm{F}^{-1}(\frac{9}{10}) - 2\mathrm{F}^{-1}(\frac{1}{2}) + \mathrm{F}^{-1}(\frac{1}{10})}{\mathrm{F}^{-1}(\frac{9}{10}) - \mathrm{F}^{-1}(\frac{1}{10})},$$

where F^{-1} is the inverse of the cumulative distribution function. Show that, for the above distribution, $D = 2 - \sqrt{5}$.

The *Pearson skewness*, P, of a distribution is defined by

$$P = \frac{3(\mu - M)}{\sigma},$$

where M is the median. Find P for the above distribution and show that $D > P > \gamma$.