

## STEP II, 2011 Q13

13 What property of a distribution is measured by its *skewness*?

(i) One measure of skewness,  $\gamma$ , is given by

$$\gamma = \frac{E((X - \mu)^3)}{\sigma^3},$$

where  $\mu$  and  $\sigma^2$  are the mean and variance of the random variable  $X$ . Show that

$$\gamma = \frac{E(X^3) - 3\mu\sigma^2 - \mu^3}{\sigma^3}.$$

The continuous random variable  $X$  has probability density function  $f$  where

$$f(x) = \begin{cases} 2x & \text{for } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Show that for this distribution  $\gamma = -\frac{2\sqrt{2}}{5}$ .

(ii) The *decile skewness*,  $D$ , of a distribution is defined by

$$D = \frac{F^{-1}(\frac{9}{10}) - 2F^{-1}(\frac{1}{2}) + F^{-1}(\frac{1}{10})}{F^{-1}(\frac{9}{10}) - F^{-1}(\frac{1}{10})},$$

where  $F^{-1}$  is the inverse of the cumulative distribution function. Show that, for the above distribution,  $D = 2 - \sqrt{5}$ .

The *Pearson skewness*,  $P$ , of a distribution is defined by

$$P = \frac{3(\mu - M)}{\sigma},$$

where  $M$  is the median. Find  $P$  for the above distribution and show that  $D > P > \gamma$ .



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