

## STEP II, 2011 Q12

- 12 Xavier and Younis are playing a match. The match consists of a series of games and each game consists of three points.

Xavier has probability  $p$  and Younis has probability  $1 - p$  of winning the first point of any game. In the second and third points of each game, the player who won the previous point has probability  $p$  and the player who lost the previous point has probability  $1 - p$  of winning the point. If a player wins two consecutive points in a single game, the match ends and that player has won; otherwise the match continues with another game.

- (i) Let  $w$  be the probability that Younis wins the match. Show that, for  $p \neq 0$ ,

$$w = \frac{1 - p^2}{2 - p}.$$

Show that  $w > \frac{1}{2}$  if  $p < \frac{1}{2}$ , and  $w < \frac{1}{2}$  if  $p > \frac{1}{2}$ . Does  $w$  increase whenever  $p$  decreases?

- (ii) If Xavier wins the match, Younis gives him  $\mathcal{L}1$ ; if Younis wins the match, Xavier gives him  $\mathcal{L}k$ . Find the value of  $k$  for which the game is fair in the case when  $p = \frac{2}{3}$ .
- (iii) What happens when  $p = 0$ ?



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