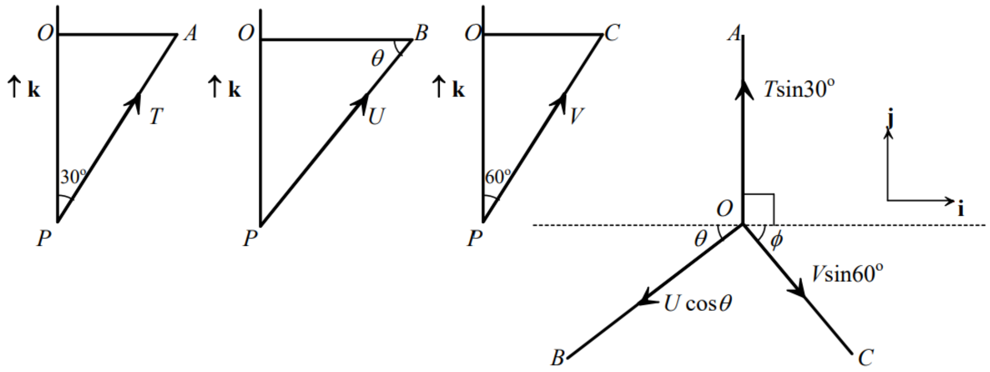
STEP II, 2011, Q11 MS

Q11 **(i)** The saying goes that "a picture paints a thousand words" and tis is especially true in mechan questions, if for no better reason than it gives the solver a clear indication of angles/directions – in this case – the forces involved. The relevant diagrams are as follows:



It might also be wise to note the sines and cosines of the given angles: $\tan\theta = \sqrt{2} \Rightarrow \sin\theta =$

and $\cos\theta = \dfrac{1}{\sqrt{3}}$, and $\tan\phi = \dfrac{\sqrt{2}}{4} \Rightarrow \sin\phi = \dfrac{1}{3}$ and $\cos\phi = \dfrac{2\sqrt{2}}{3}$. Having noted these carefully is now reasonably straightforward to state that the vector in the direction of PB is

$$-(U\cos\theta)\cos\theta\,\mathbf{i} - (U\cos\theta)\sin\theta\,\mathbf{j} + U\sin\theta\,\mathbf{k} = -\frac{1}{3}\mathbf{i} - \frac{\sqrt{2}}{3}\mathbf{j} + \frac{\sqrt{2}}{\sqrt{3}}\mathbf{k}.$$

Note that the question requires you to verify that this vector has magnitude 1.

(ii) The forces involved are now readily written down …

$\underline{\mathbf{T}_B} = \left(-\frac{1}{3}\mathbf{i} - \frac{\sqrt{2}}{3}\mathbf{j} + \frac{\sqrt{2}}{\sqrt{3}}\mathbf{k}\right)U$ follows from (i)'s answer. Also,

$\underline{\mathbf{T}_A} = T\sin 30°\,\mathbf{j} + T\cos 30°\,\mathbf{k} = \frac{1}{2}T\left(\mathbf{j} + \sqrt{3}\,\mathbf{k}\right),$

$\underline{\mathbf{T}_C} = V\sin 60°\cos\phi\,\mathbf{i} - V\sin 60°\sin\phi\,\mathbf{j} + V\cos 60°\,\mathbf{k} = \frac{1}{2}V\left(\frac{2\sqrt{2}}{\sqrt{3}}\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j} + \mathbf{k}\right)$

and $\underline{\mathbf{W}} = -W\,\mathbf{k}$.

(iii) Having set the system up in vector form, the fundamental Statics principle involved is that

$$\underline{\mathbf{T}_A} + \underline{\mathbf{T}_B} + \underline{\mathbf{T}_C} + \underline{\mathbf{W}} = \underline{\mathbf{0}}.$$

Comparing components in this vector equation gives

($\underline{\mathbf{i}}$) $0 - \frac{1}{3}U + \frac{\sqrt{6}}{3}V = 0 \Rightarrow U = V\sqrt{6}$

(**j**) $\frac{1}{2}T - \frac{\sqrt{2}}{3}U - \frac{\sqrt{3}}{6}V = 0 \Rightarrow$ (using $U = V\sqrt{6}$) $T = \frac{5\sqrt{3}}{3}V$

(**k**) $\frac{\sqrt{3}}{2}T + \frac{\sqrt{6}}{3}U + \frac{1}{2}V = W \Rightarrow$ (using $U = V\sqrt{6}$ and $T = \frac{5\sqrt{3}}{3}V$)

$$T = \frac{W\sqrt{3}}{3},\quad U = \frac{W\sqrt{6}}{5},\quad V = \frac{W}{5}.$$