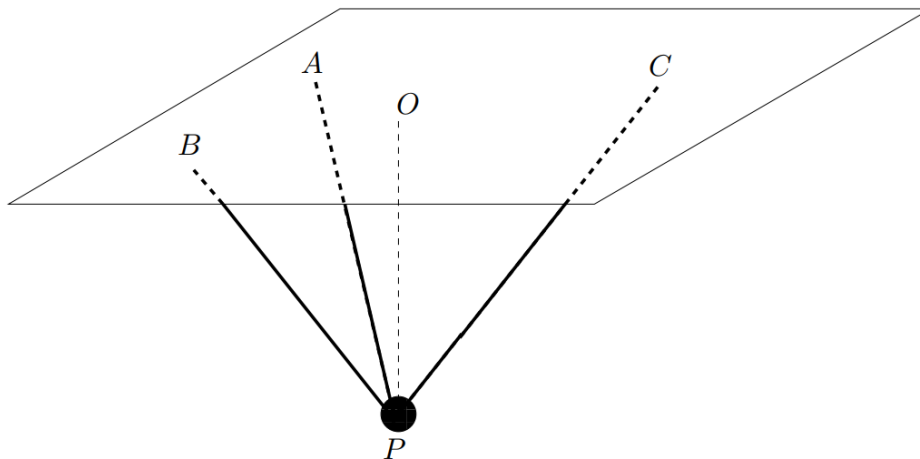


## STEP II, 2011 Q11

- 11 Three non-collinear points  $A$ ,  $B$  and  $C$  lie in a horizontal ceiling. A particle  $P$  of weight  $W$  is suspended from this ceiling by means of three light inextensible strings  $AP$ ,  $BP$  and  $CP$ , as shown in the diagram. The point  $O$  lies vertically above  $P$  in the ceiling.



The angles  $AOB$  and  $AOC$  are  $90^\circ + \theta$  and  $90^\circ + \phi$ , respectively, where  $\theta$  and  $\phi$  are acute angles such that  $\tan \theta = \sqrt{2}$  and  $\tan \phi = \frac{1}{4}\sqrt{2}$ .

The strings  $AP$ ,  $BP$  and  $CP$  make angles  $30^\circ$ ,  $90^\circ - \theta$  and  $60^\circ$ , respectively, with the vertical, and the tensions in these strings have magnitudes  $T$ ,  $U$  and  $V$  respectively.

- (i) Show that the unit vector in the direction  $PB$  can be written in the form

$$-\frac{1}{3}\mathbf{i} - \frac{\sqrt{2}}{3}\mathbf{j} + \frac{\sqrt{2}}{\sqrt{3}}\mathbf{k},$$

where  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$  are the usual mutually perpendicular unit vectors with  $\mathbf{j}$  parallel to  $OA$  and  $\mathbf{k}$  vertically upwards.

- (ii) Find expressions in vector form for the forces acting on  $P$ .
- (iii) Show that  $U = \sqrt{6}V$  and find  $T$ ,  $U$  and  $V$  in terms of  $W$ .



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