

## STEP II, 2010, Q7

- 7 (i) By considering the positions of its turning points, show that the curve with equation

$$y = x^3 - 3qx - q(1 + q),$$

where  $q > 0$  and  $q \neq 1$ , crosses the  $x$ -axis once only.

- (ii) Given that  $x$  satisfies the cubic equation

$$x^3 - 3qx - q(1 + q) = 0,$$

and that

$$x = u + q/u,$$

obtain a quadratic equation satisfied by  $u^3$ . Hence find the real root of the cubic equation in the case  $q > 0$ ,  $q \neq 1$ .

- (iii) The quadratic equation

$$t^2 - pt + q = 0$$

has roots  $\alpha$  and  $\beta$ . Show that

$$\alpha^3 + \beta^3 = p^3 - 3qp.$$

It is given that one of these roots is the square of the other. By considering the expression  $(\alpha^2 - \beta)(\beta^2 - \alpha)$ , find a relationship between  $p$  and  $q$ . Given further that  $q > 0$ ,  $q \neq 1$  and  $p$  is real, determine the value of  $p$  in terms of  $q$ .



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