STEP II, 2010, Q5

5 The points A and B have position vectors $\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $5\mathbf{i}-\mathbf{j}-\mathbf{k}$, respectively, relative to the origin O. Find $\cos 2\alpha$, where 2α is the angle $\angle AOB$.

(i) The line L_1 has equation $\mathbf{r} = \lambda(m\mathbf{i}+n\mathbf{j}+p\mathbf{k})$. Given that L_1 is inclined equally to OA and to OB, determine a relationship between m, n and p. Find also values of m, n and p for which L_1 is the angle bisector of $\angle AOB$.

(ii) The line L_2 has equation $\mathbf{r} = \mu(u\mathbf{i}+v\mathbf{j}+w\mathbf{k})$. Given that L_2 is inclined at an angle α to OA, where $2\alpha = \angle AOB$, determine a relationship between u, v and w.

Hence describe the surface with Cartesian equation $x^2+y^2+z^2 = 2(yz+zx+xy)$.