

## STEP II, 2010, Q5

- 5 The points  $A$  and  $B$  have position vectors  $\mathbf{i} + \mathbf{j} + \mathbf{k}$  and  $5\mathbf{i} - \mathbf{j} - \mathbf{k}$ , respectively, relative to the origin  $O$ . Find  $\cos 2\alpha$ , where  $2\alpha$  is the angle  $\angle AOB$ .
- (i) The line  $L_1$  has equation  $\mathbf{r} = \lambda(m\mathbf{i} + n\mathbf{j} + p\mathbf{k})$ . Given that  $L_1$  is inclined equally to  $OA$  and to  $OB$ , determine a relationship between  $m$ ,  $n$  and  $p$ . Find also values of  $m$ ,  $n$  and  $p$  for which  $L_1$  is the angle bisector of  $\angle AOB$ .
- (ii) The line  $L_2$  has equation  $\mathbf{r} = \mu(u\mathbf{i} + v\mathbf{j} + w\mathbf{k})$ . Given that  $L_2$  is inclined at an angle  $\alpha$  to  $OA$ , where  $2\alpha = \angle AOB$ , determine a relationship between  $u$ ,  $v$  and  $w$ .  
Hence describe the surface with Cartesian equation  $x^2 + y^2 + z^2 = 2(yz + zx + xy)$ .



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