

STEP II, 2010, Q4

4 (i) Let

$$I = \int_0^a \frac{f(x)}{f(x) + f(a-x)} dx.$$

Use a substitution to show that

$$I = \int_0^a \frac{f(a-x)}{f(x) + f(a-x)} dx$$

and hence evaluate I in terms of a .

Use this result to evaluate the integrals

$$\int_0^1 \frac{\ln(x+1)}{\ln(2+x-x^2)} dx \quad \text{and} \quad \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin(x + \frac{\pi}{4})} dx.$$

(ii) Evaluate

$$\int_{\frac{1}{2}}^2 \frac{\sin x}{x(\sin x + \sin \frac{1}{x})} dx.$$



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